UNIT

1

SQUARES, SQUARE ROOTS, CUBES AND CUBE ROOTS

Unit outcomes

After completing this unit, you should be able to:

- understand the notion square and square roots and cubes and cube roots.
- determine the square roots of the perfect square numbers.
- extract the approximate square roots of numbers by using the numerical table.
- determine cubes of numbers.
- extract the cube roots of perfect cubes.

Introduction

What you had learnt in the previous grade about multiplication will be used in this unit to describe special products known as squares and cubes of a given numbers. You will also learn what is meant by square roots and cube roots and how to compute them. What you will learn in this unit are basic and very important concepts in mathematics. So get ready and be attentive!

1.1 The Square of a Number

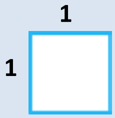
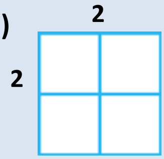
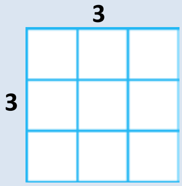
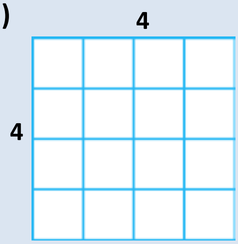
1.1.1 Square of a Rational Number

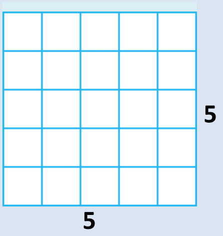
Addition and subtraction are operations of the first kind while multiplication and division are operation of the second kind. Operations of the third kind are **raising to a power** and **extracting roots**. In this unit, you will learn about raising a given number to the power of "2" and power of "3" and extracting square roots and cube roots of some perfect squares and cubes.

Group Work 1.1

Discuss with your friends

1. Complete this Table 1.1. Number of small squares

	Standard Form	Factor Form	Power Form
a) 	1	1×1	1^2
b) 	4	2×2	2^2
c) 	____________	____________	____________
d) 	____________	____________	____________

e)  Figure 1.1	____________	____________	____________

2. Put three different numbers in the circles so that when you add the numbers at the end of each – line you always get a square number.

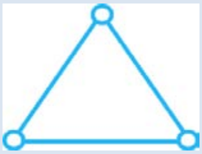


Figure 1.2

3. Put four different numbers in the circles so that when you add the numbers at the end of each line you always get a square number.

Figure 1.3

Definition 1.1: The process of multiplying a rational number by itself is called squaring the number.

For example some few square numbers are:

a) $1 \times 1 = 1$ is the 1st square number. c) $3\times 3 = 9$ is the 3rd square number.

b) $2\times 2 = 4$ is the 2nd square number. d) $4\times4 = 16$ is the 4th square number.

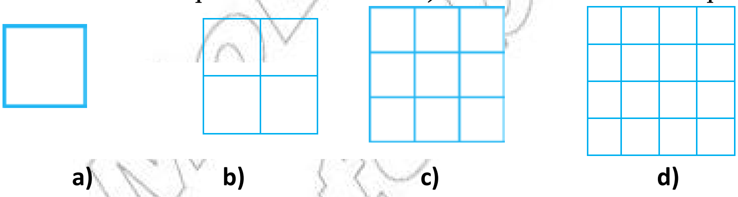

a) b) c) d)

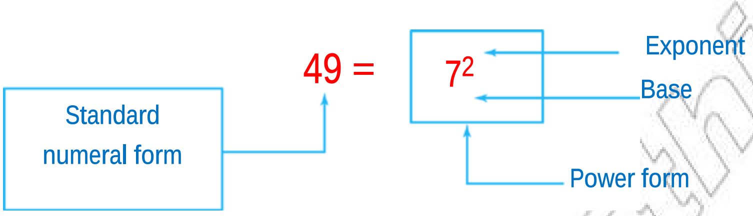
Figure 1.4 A square number can be shown as a pattern of squares

If the number to be multiplied by itself is ‘a’, then the product (or the result a × a) is usually written as a^2 and is read as:

- ✓ a squared or
- ✓ the square of a or
- ✓ a to the power of 2

In geometry, for example you have studied that the area of a square of side length ‘a’ is a × a or briefly a^2.

When the same number is used as a factor for several times, you can use an exponent to show how many times this numbers is taken as a factor or base.



Note: 7^2 is read as

- ✓ 7 squared or
- ✓ the square of 7 or
- ✓ 7 to the power of 2

Example 1: Find the square of each of the following.

a) 8 b) 10 c) 14 d) 19

Solution

a) $8^2 = 8 \times 8 = 64$

b) $10^2 = 10 \times 10 = 100$

c) $14^2 = 14 \times 14 = 196$

d) $19^2 = 19 \times 19 = 361$

Example2: Identify the base, exponent, power form and standard form of the following expression.

a) 10^2 b) 18^2

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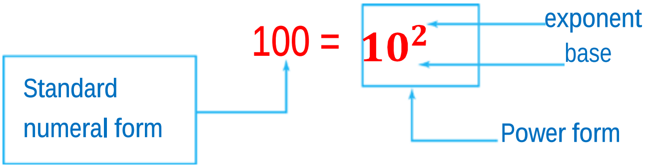
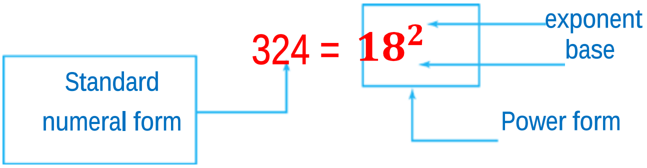
Solution

a)



b)



Note: There is a difference between a^2 and $2a$. To see this distinction consider the following examples of comparison.

Example3: a) $30^2 = 30 \times 30 = 900$ while $2 \times 30 = 60$

b) $40^2 = 40 \times 40 = 1600$ while $2 \times 40 = 80$

c) $52^2 = 52 \times 52 = 2704$ while $2 \times 52 = 104$

Hence from the above example; you can generalize that $a^2 = a \times a$ and $2a = a + a$, are quite different expressions.

Definition 1.2: A rational number x is called a perfect square, if and only if $x = n^2$ for some $n \in \mathbb{Q}$.

Example 4: $1 = 1^2$, $4 = 2^2$, $9 = 3^2$, $16 = 4^2$, $25 = 5^2$. Thus 1, 4, 9, 16 and 25 are perfect squares.

Note: A perfect square is a number that is a product of a rational number times itself and its square root is a rational number.

Example5: In Table 1.2 below some natural numbers are given as values of x. Find x^2 and complete table 1.2.

x	1	2	3	4	5	10	15	20	25	35
x^2										

Solution

When $x = 1, x^2 = 1^2 = 1 \times 1 = 1$

When $x = 2, x^2 = 2^2 = 2 \times 2 = 4$

When $x = 3, x^2 = 3^2 = 3 \times 3 = 9$

When $x = 4, x^2 = 4^2 = 4 \times 4 = 16$

When $x = 5, x^2 = 5^2 = 5 \times 5 = 25$

When $x = 10, x^2 = 10^2 = 10 \times 10 = 100$

When $x = 15, x^2 = 15^2 = 15 \times 15 = 225$

When $x = 20, x^2 = 20^2 = 20 \times 20 = 400$

When $x = 25, x^2 = 25^2 = 25 \times 25 = 625$

When $x = 35, x^2 = 35^2 = 35 \times 35 = 1225$

x	1	2	3	4	5	10	15	20	25	35
x^2	1	4	9	16	25	100	225	400	625	1225

You have so far been able to recognize the squares of natural numbers, you also know that multiplication is closed in the set of rational numbers. Hence it is possible to multiply any rational number by itself.

Example 6: Find x^2 in each of the following where x is rational number given as:

a) $x = \frac{4}{3}$ b) $x = \frac{1}{3}$ c) $x = \frac{3}{5}$ d) $x = 0.26$

Solution

a) $x^2 = \left(\frac{4}{3}\right)^2 = \frac{4}{3} \times \frac{4}{3} = \frac{4 \times 4}{3 \times 3} = \frac{16}{9}$

b) $x^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{3} \times \frac{1}{3} = \frac{1 \times 1}{3 \times 3} = \frac{1}{9}$

c) $x^2 = \left(\frac{3}{5}\right)^2 = \frac{3}{5} \times \frac{3}{5} = \frac{3 \times 3}{5 \times 5} = \frac{9}{25}$

d) $x^2 = (0.26)^2 = \left(\frac{26}{100}\right)^2 = \frac{26}{100} \times \frac{26}{100} = \frac{26 \times 26}{100 \times 100} = \frac{676}{10,000}$

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Note:

i. The squares of natural numbers are also natural numbers.

ii. $0 \times 0 = 0$ therefore $0^2 = 0$

iii. We give no meaning to the symbol 0^0

iv. If $a \in \mathbb{Q}$ and $a \neq 0$, then $a^0 = 1$

v. For any rational number 'a', $a \times a$ is denoted by a^2 and read as "a squared" or "a to the power of 2" or "the square of a".

Exercise 1A

1. Determine whether each of the following statements is true or false.

   a) $15^2 = 15 \times 15$

   b) $20^2 = 20 \times 20$

   c) $19^2 = 19 \times 19$

   d) $81^2 = 2\times 81$

   e) $41 \times 41 = 41^2$

   f) $-(50)^2 = 2500$

   g) $x^2 = 2^x$

   h) $x^2 = 2^{2x}$

   i) $(-60)^2 = 3600$

2. Complete the following.

   a) $12 \times$ _____ $= 144$

   b) $51 \times$ _____ $= 2601$

   c) $60^2 =$ ____ $\times$ ____

   d) $(3a)^2 =$ ___ $\times$ ___

   e) $8a =$ ___ + ____

   f) $28 \times 28 =$ ______

3. Find the square of each of the following.

   a) 8 b) 12 c) 19 d) 51 e) 63 f) 100

4. Find x^2 in each of the following.

   a) $x = 6$

   b) $x = \frac{1}{6}$

   c) $x = -0.3$

   d) $x = -20$

   e) $x = \frac{-50}{3}$

   f) $x = 56$

   g) $x = 0.07$

5. a. write down a table of square numbers from the first to the tenth.

   b. Find two square numbers which add to give a square number.

6. Explain whether:

   a. 441 is a square number.

   b. 2001 is a square number.

   c. 1007 is a square number.

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Challenge Problems

7. Find

   a) The 8th square number.

   b) The 12th square number.

   c) The first 12 square numbers.

8. From the list given below indicate all numbers that are perfect squares.

   a) 50 20 64 30 1 80 8 49 9

   b) 10 21 57 4 60 125 7 27 48 16 25 90

   c) 137 150 75 110 50 625 64 81 144

   d) 90 180 216 100 81 75 140 169 125

9. Show that the difference between any two consecutive square numbers is an odd number.

10. Show that the difference between the 7th square number and the 4th square number is a multiple of 3.

Theorem1.1: Existence theorem

For each rational number x, there is a rational number y ($y \geq 0$) such that $x^2 = y$.

Example 7: By the existence theorem, if

a) $x = 9$, then $y = 9^2 = 81$

b) $x = 0.5$, then $y = (0.5)^2 = 0.25$

c) $x = -17$, then $y = (-17)^2 = 289$

d) $x = \frac{7}{11}$, then $y = \left(\frac{7}{11}\right)^2 = \frac{49}{121}$

Rough calculation could be carried out for approximating and checking the results in squaring rational numbers. Such an approximation depends on rounding off decimal numbers as it will be seen from the following examples.

Examples 8: Find the approximate values of x^2 in each of the following:

a) $x = 3.4$

b) $x = 9.7$

c) $x = 0.026$

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Solution

a) $3.4 \approx 3$ thus $(3.4)^2 \approx 3^2 = 9$

b) $9.7 \approx 10$ thus $(9.7)^2 \approx 10^2 = 100$

c) $0.026 \approx 0.03$ thus $(0.026)^2 \approx \left(\frac{3}{100}\right)^2 = 0.0009$

Exercise 1B

1. Determine whether each of the following statements is true or false.
   - a) $(4.2)^2$ is between 16 and 25
   - b) $0^2 = 2$
   - c) $11^2 > (11.012)^2 > 12^2$
   - d) $(9.9)^2 = 100$
   - e) $(-13)^2 = -169$
   - f) $81 \times 27 = 9^2 \times 9 \times 3$
2. Find the approximate values of x^2 in each of the following.
   - a) $x = 3.2$
   - b) $x = 9.8$
   - c) $x = -12.1$
   - d) $x = 2.95$
   - e) $x = 0.086$
   - f) $x = 8.80$
3. Find the square of the following numbers and check your answers by rough calculation.
   - a) 0.87
   - b) 16.45
   - c) 12.12
   - d) 42. 05
   - e) 25. 14
   - f) 28. 23
   - g) 38.9
   - h) 54. 88

1.1.2 Use of Table of Values of Squares

Activity 1.1

Discuss with your friends / partners/

Use table of square to find x^2 in each of the following.

- a) $x = 1.08$
- b) $x = 2.26$
- c) $x = 9.99$
- d) $x = 1.56$
- e) $x = 5.48$
- f) $x = 7.56$

- ✓ To find the square of a rational number when it is written in the form of a decimal is tedious and time consuming work. To avoid this tedious and time consuming work a table of squares is prepared and presented in the "Numerical tables" at the end of this book.
- ✓ In this table the first column headed by x lists numbers starting from 1.0. The remaining columns are headed respectively by the digits 0 to 9.

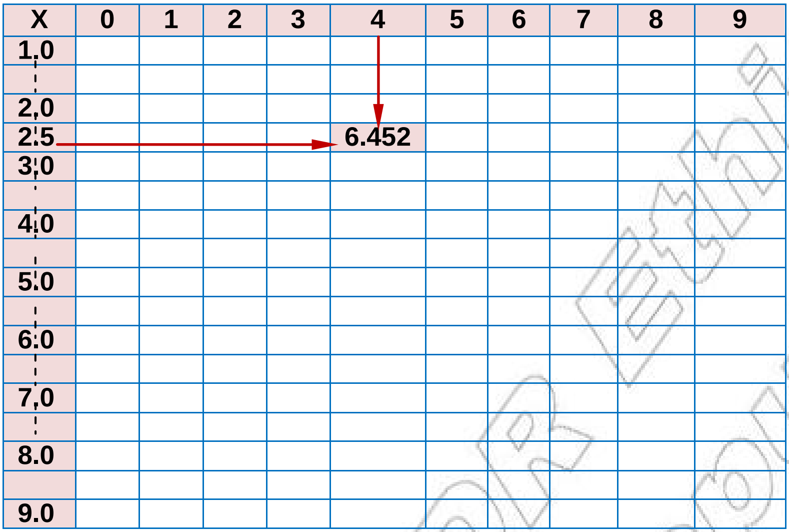
Now if you want to determine the square of a number for example 2.54 proceed as follows.

Step i. Under the column headed by x, find the row with 2.5.

Step ii. Move to the right along the row until you get the column under 4, (or find the column headed by 4).

Step iii. Then read the number at the intersection of the row in (i) and the column (ii), (see the illustration below).

Hence $(2.54)^2 = 6.452$



X	0	1	2	3	4	5	6	7	8	9
1.0										
2.0										
2.5					6.452					
3.0										
4.0										
5.0										
6.0										
7.0										
8.0										
9.0										

Figure 1.5 Tables of squares

Note that the steps (i) to (iii) are often shortened by saying “2.5 under 4”.

- ✓ Mostly the values obtained from the table of squares are only approximate values which of course serves almost for all practical purposes.

Group work 1.2

Discuss with your group.

Find the square of the number 8.95

a) use rough calculation method.

b) use the numerical table.

c) by calculating the exact value of the number.
d) compare your answer from “a” to “c”.
e) write your generalization.

Example9: Find the square of the number 4.95.

Solution: Do rough calculation and compare your answer with the value obtained from the table.

i. Rough calculation

$4.95 \approx 5$ and $5^2 = 25$

$(4.\ 95)^2 \approx 25$

ii. Value obtained from the table

i) Find the row which starts with 4.9.
ii) Find the column headed by 5.
iii) Read the number, that is $(4.\ 95)^2$ at the intersection of the row in (i) and the column in (ii) ; $(4.\ 95)^2 = 24.\ 50$.

iii. Exact Value

Multiply 4.95 by 4.95

$4.95 \times 4.95 = 24.\ 5025$

Therefore $(4.95)^2 = 24.\ 5025$.

This example shows that the result obtained from the “Numerical table” is an approximation and more closer to the exact value.

Exercise 1C

1. Determine whether each of the following statements is true or false.

a) $(2.3)^2 = 5.429$
b) $(9.1)^2 = 973.\ 2$
c) $(3.56)^2 = 30.\ 91$
d) $(9.90)^2 = 98.\ 01$
e) $(5.\ 67)^2 = 32$
f) $(4.\ 36)^2 = 16.2$

2. Find the squares of the following numbers from the table.

a) 4. 85
b) 6.46
c) 88.2
d) 29. 0
e) 2. 60
f) $\frac{3}{2}$
g) 498
h) 246
i) 165

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1.2 The Square Root of a Rational Number

Group Work 1.3

Discuss with your Friends

Find the square root of each of the following numbers.

a) 81 **b)** 324 **c)** $\frac{1}{4}$ **d)** $\frac{64}{49}$ **e)** $\frac{4}{49}$ **f)** $\frac{25}{625}$

- ✓ So far you have studied the meaning of x^2 when x is a rational number. It is now logical to ask, whether you can go in the reverse (or in the opposite direction) or not. In this sub unit you will answer this and related questions in a more systematized manner.

Definition 1.3: Square roots

For any two rational numbers a and b if $a^2 = b$, then a is called the square root of b.

Example 10:

a) 4 is the square root of 16, since $4^2 = 16$.

b) 5 is the square root of 25, since $5^2 = 25$.

c) 6 is the square root of 36, since $6^2 = 36$.

Example 11: The area of a square is 49m^2. What is the length of each side?

Solution:

$$\ell \times w = A$$

$$s \times s = 49 \text{ m}^2$$

$$s^2 = 49 \text{ m}^2$$

$$s = 7\text{m}$$

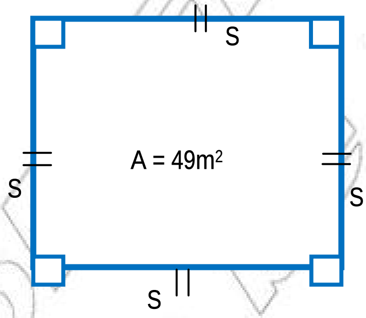

Figure 1.6

The length of each side is 7 meters. This is one way to express the mathematical relationship “7 is the square root of 49” because $7^2 = 49$.

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Note:

i. The notion "*square root*" is the inverse of the notion "*square of a number*".

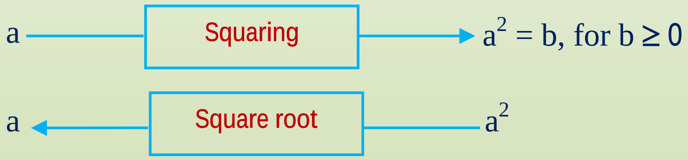
ii. The operation "*extracting square root*" is the inverse of the operation "*squaring*".

iii. In extracting square roots of rational numbers, first decompose the number into a product consisting of two equal factors and take one of the equal factors as the square root of the given number.

iv. The symbol or notation for square root is "$\sqrt{\ }$" it is called radical sign.

v. For b ≥ 0, the expression $\sqrt{b}$ is called radical b and the number b is called a radicand.

vi. The relation of squaring and square root can be expressed as follows:



vii. a is the square root of b and written as $a = \sqrt{b}$.

Example 12: Find the square root of x, if x is:

a) 100 b) 125 c) 169 d) 256 e) 625 f) 1600

Solution

a) $x = 100 = 10 \times 10$

$x = 10^2$, thus the square root of 100 is 10.

b) $x = 225 = 15 \times 15$

$x = 15^2$, thus the square root of 225 is 15.

c) $x = 169 = 13 \times 13$

$x = 13^2$, thus the square root of 169 is 13.

d) $x = 256 = 16 \times 16$

$x = 16^2$, thus the square root of 256 is 16.

e) $x = 625 = 25 \times 25$

$x = 25^2$, thus the square root of 625 is 25.

f) $x = 1600 = 40 \times 40$

$x = 40^2$, thus the square root of 1600 is 40.

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Exercise 1D

1. Determine whether each of the following statements is true or false.

   a) $\sqrt{0} = 0$

   b) $\sqrt{25} = \pm 5$

   c) $\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$

   d) $-\sqrt{121} = -11$

   e) $-\sqrt{\frac{36}{324}} = \frac{1}{3}$

   f) $\sqrt{\frac{324}{625}} = \frac{18}{25}$

   g) $-\sqrt{\frac{900}{961}} = -\frac{30}{31}$

2. Find the square root of each of the following numbers.

   a) 121

   b) 144

   c) 289

   d) 361

   e) 400

   f) 441

   g) 484

   h) 529

3. Evaluate each of the following.

   a) $\sqrt{\frac{1}{25}}$

   b) $\sqrt{\frac{1}{81}}$

   c) $-\sqrt{\frac{36}{144}}$

   d) $-\sqrt{576}$

   e) $\frac{\sqrt{529}}{\sqrt{625}}$

   f) $-\sqrt{676}$

   g) $\sqrt{729}$

   h) $-\sqrt{784}$

   i) $\sqrt{\frac{16}{25}}$

Challenge Problems

4. If $\frac{x}{y} = -2$. Find $\sqrt{\frac{x^2}{y^2} + \frac{y^2}{x^2}}$

5. Simplify: $\sqrt{(81)^2} + \sqrt{(49)^2}$

6. If $x = 16$ and $y = 625$. Find $\left(2\sqrt{x+y}\right)^2$.

Definition 1.4 : If a number $y \geq 0$ is the square of a positive number x ($x \geq 0$), then the number x is called the square root of y.

This can be written as $x = \sqrt{y}$.

Example13: Find

a) $\sqrt{0.01}$ c) $\sqrt{0.81}$ e) $\sqrt{0.7921}$ g) $\sqrt{48.8601}$

b) $\sqrt{0.25}$ d) $\sqrt{0.6889}$ f) $\sqrt{0.9025}$

Solution

a) $\sqrt{0.01} = \sqrt{0.1 \times 0.1} = 0.1$

b) $\sqrt{0.25} = \sqrt{0.5 \times 0.5} = 0.5$

c) $\sqrt{0.81} = \sqrt{0.9 \times 0.9} = 0.9$

d) $\sqrt{0.6889} = \sqrt{0.83 \times 0.83} = 0.83$

e) $\sqrt{0.7921} = \sqrt{0.89 \times 0.89} = 0.89$

f) $\sqrt{0.9025} = \sqrt{0.95 \times 0.95} = 0.95$

g) $\sqrt{48.8601} = \sqrt{6.99 \times 6.99} = 6.99$

Exercise 1E

Simplify the square roots.

a) $\sqrt{35.88}$ c) $\sqrt{89.87}$ e) $\sqrt{62.25}$

b) $\sqrt{36.46}$ d) $\sqrt{99.80}$ f) $\sqrt{97.81}$

1.2.1 Square Roots of Perfect Squares

Group work 1.4

Discuss with your group.

1. Find the prime factorization of the following numbers by using the factor trees.

a) 64 c) 121 e) 324 g) 625 i) 700

b) 81 d) 289 f) 400 h) 676

Note: The following properties of squares are important:

$$(ab)^2 = a^2 \times b^2 \text{ and } \left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2} \text{(where } b \neq 0\text{).}$$

$$\text{Thus } (2\times3)^2 = 2^2 \times 3^2 = 36 \text{ and } \left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16}.$$

Remember a number is called a perfect square, if it is the square of a rational number.

The following properties are useful to simplify square roots of numbers.

Properties of Square roots, for $a \geq 0$, $b \geq 0$.

If $\sqrt{a}$ and $\sqrt{b}$ represent rational numbers, then

$$\sqrt{ab} = \sqrt{a}\sqrt{b} \text{ and } \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \text{ where } b \neq 0.$$

Example: 14 Determine whether each of the following numbers is a perfect square or not.

a) 36 c) 81 e) $\frac{16}{625}$ g) 11

b) 49 d) $\frac{49}{25}$ f) 7

Solution: a) 36 is a perfect square, because $36 = 6^2$.

b) 49 is a perfect square, because $49 = 7^2$.

c) 81 is a perfect square, because $81 = 9^2$.

d) $\frac{49}{25}$ is a perfect square, becaus $\frac{49}{25} = \left(\frac{7}{5}\right)^2$.

e) $\frac{16}{625}$ is a perfect square, because $\frac{16}{625} = \left(\frac{4}{25}\right)^2$.

f) 7 is not a perfect square since there is no rational number whose square is equal to 7. In other words there is no rational number n such that $n^2 = 7$.

g) 11 is not a perfect square since there is no rational number whose square is equal to 11. In short there is no rational number n such that $n^2 = 11$.

Example 15: Use prime factorization and find the square root of each of the following numbers.

a) $\sqrt{324}$ b) $\sqrt{400}$ c) $\sqrt{484}$

Solution: a) $324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$

Now arrange the factors so that 324 is a product of two identical sets of prime factors.

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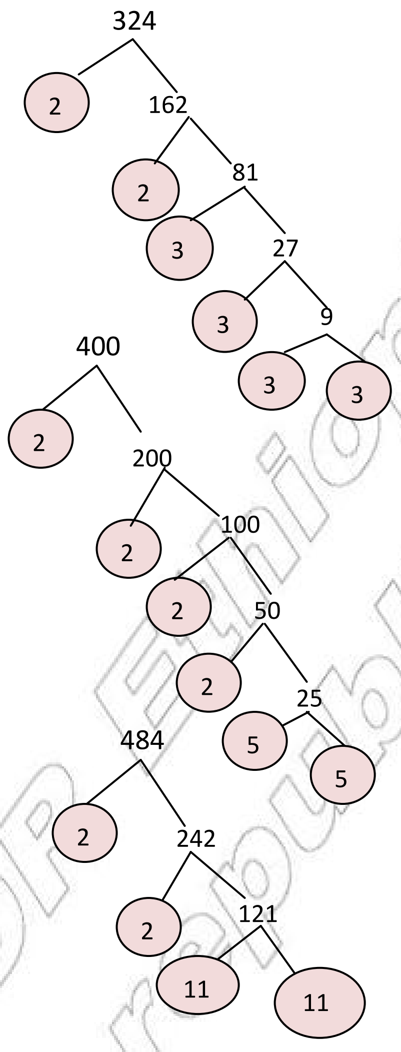
i.e $324 = (2 \times 2 \times 3\times3 \times 3\times3)$

$= (2 \times 3 \times 3) \times (2 \times 3 \times 3)$

$= 18 \times 18 = 18^2$

So, $\sqrt{324} = \sqrt{18\times18} = 18$



b) $400 = (2 \times 2 \times 5) \times (2 \times 2 \times 5)$

Now arrange the factors so that 400 is a product of two identical sets of prime factors.

i.e $400 = (2\times 2 \times 5)\times(2 \times 2\times5)$

$= 20 \times 20$

$= 20^2$

So $\sqrt{400} = \sqrt{20\times20}$

$= 20$

c) $484 = 2\times2\times 11\times11$, now arrange the factors so that 484 is a product of two identical sets of prime factors.

i.e $484 = 2\times2 \times 11\times11$

$= (2\times 11)\times (2 \times 11)$

$= 22 \times 22 = 22^2$

So $\sqrt{484} = \sqrt{22\times22}$

$= 22$

Exercise 1F

1. Determine whether each of the following statements is true or false.

a) $\sqrt{64\times25} = \sqrt{64}\times\sqrt{25}$

b) $\sqrt{\frac{64}{4}} = 4$

c) $\sqrt{\frac{32}{64}} = \frac{\sqrt{32}}{\sqrt{64}}$

d) $\sqrt{\frac{0}{1296}} = 0$

e) $\sqrt{\frac{1296}{0}} = 1$

f) $\sqrt{\frac{729}{1444}} = \frac{27}{38}$

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2. Evaluate each of the following.

a) $\sqrt{0.25}$ c) $\sqrt{\frac{1296}{1024}}$ e) $\sqrt{\frac{81}{324}}$

b) $\sqrt{0.0625}$ d) $\sqrt{\frac{625}{1024}}$ f) $\sqrt{\frac{144}{400}}$

Challenge Problem

3. Simplify a) $\sqrt{625 - 0} - \sqrt{172 - 3}$

b) $\sqrt{81 \times 625}$

c) $\sqrt{\left(\frac{1}{64}\right)^2}$

4. Does every number have two square roots? Explain.

5. Which of the following are perfect squares?

{0, 1, 4, 7, 12, 16, 25, 30, 36, 42, 49}

6. Which of the following are perfect squares?

{50, 64, 72, 81, 95, 100, 121, 140, 144, 169}

7. Copy and complete.

a) $3^2 + 4^2 + 12^2 = 13^2$

b) $5^2 + 6^2 +$ ____ = ______

c) $6^2 + 7^2 +$ _______ = ______

d) $x^2 + (x + 1)^2 +$ ____ = ____

Using the square root table

The same table which you can use to determine squares of numbers can be used to find the approximate square roots, of numbers.

Example 16: Find $\sqrt{17.89}$ from the numerical table.

Solution:

Step i. Find the number 17.89 in the body of the table for the function $y = x^2$.

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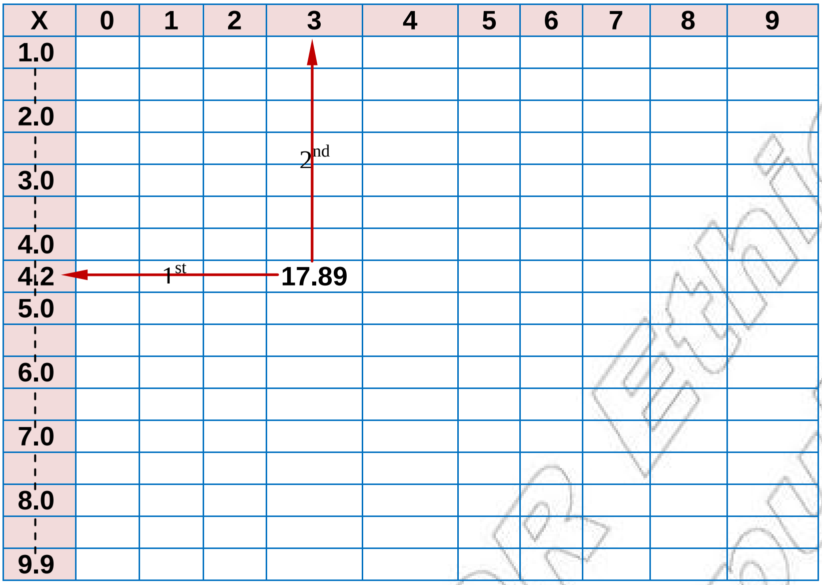
Step ii. On the row containing this number move to the left and read 4.2 under x.

These are the first two digits of the square root of 17. 89

Step iii. To get the third digit start from 17.89 move vertically up ward and read 3.

Therefore $\sqrt{17.89} \approx 4.23$



X	0	1	2	3	4	5	6	7	8	9
1.0										
2.0										
3.0										
4.0										
4.2				17.89						
5.0										
6.0										
7.0										
8.0										
9.9										

Figure 1.7 Table of square roots

If the radicand is not found in the body of the table, you can consider the number which is closer to it.

Example 17: Find $\sqrt{10.59}$

Solution: i) It is not possible to find the number 10. 59 directly in the table of squares. But in this case find two numbers in the table which are closer to it, one from left (i.e. 10.56) and one form right (10.63) that means $10.56 < 10.59 < 10.63$.

ii) Find the nearest number to (10.59) form those two numbers. So the nearest number is 10.56 thus $\sqrt{10.59} \approx \sqrt{10.56} = 3.25$.

Example 18: Find $\sqrt{83.60}$

Solution:

i. It is not possible to find the number 83. 60 directly in the table of squares. But find two numbers which are closer to it, one from left (i.e. 83.54) and one from right (i.e 83.72) that means $83.54 < 83.60 < 83.72$.

ii. Find the nearest number from these two numbers. Therefore the nearest number is 83. 54, so $\sqrt{83.60} \approx \sqrt{83.54} = 9.14$.

Note: To find the square root of a number greater than 100 you can use the method illustrated by the following example.

Example 19: Find the square root of each of the following.

a) $\sqrt{6496}$ b) $\sqrt{9801}$ c) $\sqrt{9880}$ d) $\sqrt{9506}$

Solution:

a) $\sqrt{6496} = \sqrt{64.96\times100}$
$= \sqrt{64.96} \times \sqrt{100}$
$= 8.06\times10$
$= 80.6$

c) $\sqrt{9880} = \sqrt{98.80\times100}$
$= \sqrt{98.80} \times \sqrt{100}$
$= 9.94\times10$
$= 99.4$

b) $\sqrt{9801} = \sqrt{98.01\times100}$
$= \sqrt{98.01} \times \sqrt{100}$
$= 9.90\times10$
$= 99$

d) $\sqrt{9506} = \sqrt{95.06\times100}$
$= \sqrt{95.06} \times \sqrt{100}$
$= 9.75\times10$
$= 97.5$

Example 20: Find the square root of each of the following numbers by using the table

a) $\sqrt{98.41}$ c) $\sqrt{984100}$ e) $\sqrt{0.009841}$

b) $\sqrt{9841}$ d) $\sqrt{0.9841}$ f) $\sqrt{0.00009841}$

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Solution:

a) $\sqrt{98.41} = 9.92$

b) $\sqrt{9841} = \sqrt{98.41 \times 100}$

$= \sqrt{98.41} \times \sqrt{100}$

$= 9.92 \times 10$

$= 99.2$

c) $\sqrt{984100} = \sqrt{98.41 \times 10000}$

$= \sqrt{98.41} \times \sqrt{10000}$

$= 9.92 \times 100$

$= 992$

d) $\sqrt{0.9841} = \sqrt{98.41 \times \frac{1}{100}}$

$= \sqrt{98.41} \times \sqrt{\frac{1}{100}}$

$= 9.92 \times \frac{1}{10}$

$= 0.992$

e) $\sqrt{0.009841} = \sqrt{98.41 \times \frac{1}{10{,}000}}$

$= \sqrt{98.41} \times \sqrt{\frac{1}{10{,}000}}$

$= 9.92 \times \frac{1}{100}$

$= 0.0992$

f) $\sqrt{0.00009841} = \sqrt{98.41} \times \sqrt{\frac{1}{1{,}000{,}000}}$

$= 9.92 \times \frac{1}{1{,}000}$

$= 0.00992$

Exercise 1G

1. Find the square root of each of the following numbers from the table.

a) 15.37
b) 40.70
c) 121.3
d) 153.1
e) 162.8
f) 163.7
g) 997
h) 6034
i) 6076
j) 5494
k) 5295
l) 3874

2. Use the table of squares to find approximate value of each of the following.

a) $\sqrt{6.553}$
b) $\sqrt{8.761}$
c) $\sqrt{24.56}$
d) $\sqrt{29.78}$

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1.3 Cubes and Cube Roots

1.3.1 Cube of a Number

If the number to be cubed is ‘a’, then the product $a \times a \times a$ which is usually written as a^3 and is read as ‘a’ cubed. For example 3 cubed gives 27 because $3 \times 3 \times 3 = 27$.

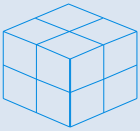
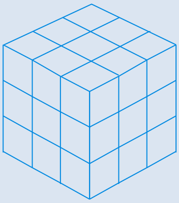
The product 3×3×3 can be written as 3^3 and which is read as 3 cubed.

Activity 1.2

Discuss with your friends

1. Copy and complete this Table 1.3

Number of small cubes

	Standard form	Factor form	Power form
a) 	1	1 × 1 × 1	1^3
b) 	8	2 × ____ × ____	2^3
c)  Figure 1.8	27	____ × ____ × ____	$\square^{\square}$

2. a) Which of these numbers are cubic numbers?

64 100 125 216 500 1000
1728 3150 4096 8000 8820 15625

b) Write the cubic numbers from part (a) in power form.

3. Find a^3 in each of the following.

a) a = 2 c) a = 10 e) a = 0.5
b) a = -2 d) $a = \frac{1}{4}$ f) a = 0.25

Definition 1.5: A cube number is the result of multiplying a rational number by itself, then multiplying by the number again.

For example, some few cube numbers are:

a) $1 \times 1 \times 1 = 1$ is the 1st cube number.

b) $2 \times 2 \times 2 = 8$ is the 2nd cube number.

c) $3 \times 3 \times 3 = 27$ is the 3rd cube number.

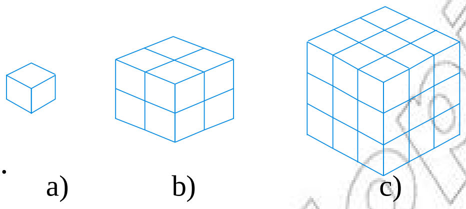


Figure 1.9 A cube number can be shown as a pattern of cubes

Example 21: Find the numbers whose cube are the following.

a) 4,913 b) 6,859 c) 9,261 d) 29,791

Solution:

a) $4{,}913 = 17 \times 17 \times 17 = 17^3$

b) $6{,}859 = 19 \times 19 \times 19 = 19^3$

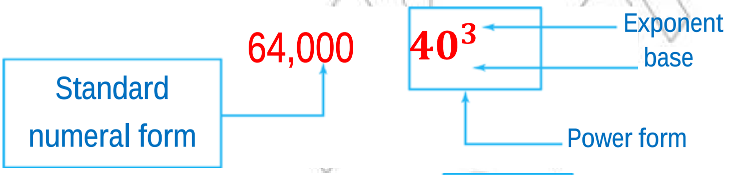
c) $9{,}261 = 21 \times 21 \times 21 = 21^3$

d) $29{,}791 = 31 \times 31 \times 31 = 31^3$

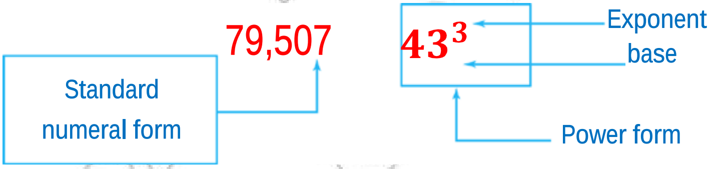
Example 22: Identify the base, exponent, power form and standard numeral form:

a) 40^3 b) 43^3

Solution: a)



b)



Example 23: In Table1.4 below integers are as values of x, find x^3 and complete the table 1.4.

x	-4	-3	-2	-1	0	1	2	3	4	5	6
x^3											

Solution:

When $x = -4$, $x^3 = (-4)^3 = -4 \times -4 \times -4 = -64$
When $x = -3$, $x^3 = (-3)^3 = -3 \times -3 \times -3 = -27$
When $x = -2$, $x^3 = (-2)^3 = -2 \times -2 \times -2 = -8$
When $x = -1$, $x^3 = (-1)^3 = -1 \times -1 \times -1 = -1$
When $x = 0$, $x^3 = 0^3 = 0 \times 0 \times 0 = 0$
When $x = 1$, $x^3 = 1^3 = 1 \times 1 \times 1 = 1$
When $x = 2$, $x^3 = 2^3 = 2 \times 2 \times 2 = 8$
When $x = 3$, $x^3 = 3^3 = 3 \times 3 \times 3 = 27$
When $x = 4$, $x^3 = 4^3 = 4 \times 4 \times 4 = 64$
When $x = 5$, $x^3 = 5^3 = 5 \times 5 \times 5 = 125$
When $x = 6$, $x^3 = 6^3 = 6 \times 6 \times 6 = 216$

Lastly you have:

x	-4	-3	-2	-1	0	1	2	3	4	5	6
x^3	-64	-27	-8	-1	0	1	8	27	64	125	216

The examples above illustrate the following theorem. This theorem is called **existence theorem.**

Theorem 1.2: Existence theorem

For each rational number x, there is a rational number y such that $y = x^3$.

Rough calculations could be used for approximating and checking the results in cubing rational numbers. The following examples illustrate the situation.

Example 24: Find the approximate values of x^3 in each of the following.

a) $x = 2.2$ b) $x = 0.065$ c) $x = 9.54$

Solution:

a. $2.2 \approx 2$ thus $(2.2)^3 \approx 2^3 = 8$

b. $0.065 \approx 0.07$ thus $(0.065)^3 \approx \left(\frac{7}{100}\right)^3 = \frac{343}{1{,}000{,}000}$
$= 0.000343$

c. $9.54 \approx 10$ thus $(9.54)^3 \approx 10^3 = 1{,}000$

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Exercise 1H

1. Determine whether each of the following statements is true or false.

a) $4^3 = 16 \times 4$

b) $4^3 = 64$

c) $(-3)^3 = 27$

d) $\left(\frac{3}{4}\right)^3 = \frac{27}{16}$

e) $\left(\frac{4}{3}\right)^3 = \frac{64}{125}$

f) $\sqrt[3]{64} = 4$

2. Find x^3 in each of the following.

a) $x = 8$

b) $x = 0.4$

c) $x = -4$

d) $x = -\frac{1}{4}$

e) $x = \frac{-1}{5}$

f) $x = -0.\ 2$

3. Find the approximate values of x^3 in each of the following.

a) $x = -2.49$

b) $x = 2.29$

c) $x = 2.98$

d) $x = 0.025$

Challenge Problem

4. The dimensions of a cuboid are 4xcm, 6xcm and 10xcm. Find
   a) The total surface area
   b) The volume

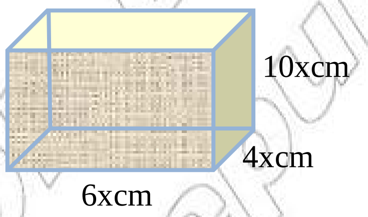


Figure 1.10 Cuboid

Table of Cubes

Activity 1.3

Discuss with your friends

Use the table of cubes to find the cubes of each of the following.

a) 2.26
b) 5.12
c) 5.99
d) 8.48
e) 8.86
f) 9.48
g) 9.58
h) 9.89
i) 9.99
j) 9.10

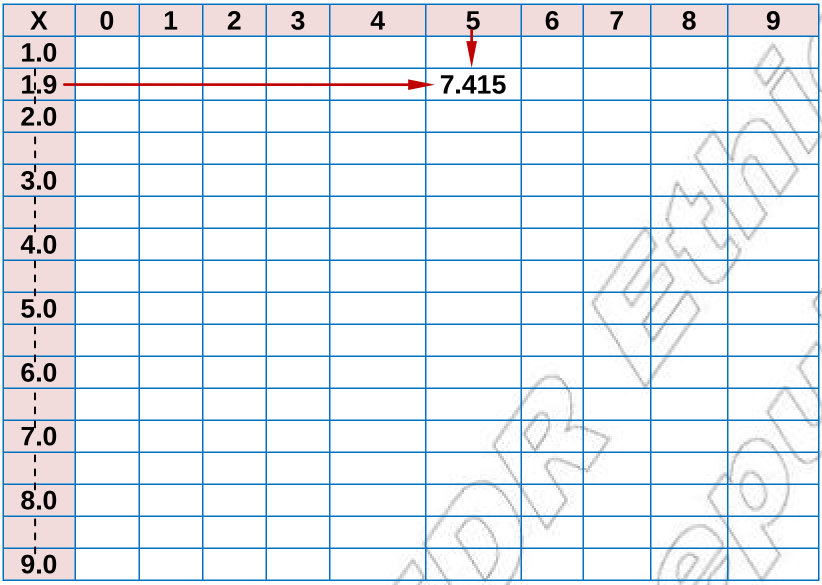
To find the cubes of a rational number when it is written in the form of a decimal is tedious and time consuming work. To avoid this tedious and time consuming work, a table of cubes is prepared and presented in the "Numerical Tables" at the end of this textbook.

In this table the first column headed by ‘x’ lists numbers starting from 1.0. The remaining columns are headed respectively by the digit 0 to 9.0. Now if we want to determine the cube of a number, for example 1.95 Proceed as follows.

Step i. Find the row which starts with 1.9 (or under the column headed by x).

Step ii. Move to the right until you get the number under column 5 (or find the column headed by 5).

Step iii. Then read the number at the intersection of the row in step (i) and the column step (ii) therefore we find that $(1.95)^3 = 7.415$. See the illustration below.



X	0	1	2	3	4	5	6	7	8	9
1.0										
1.9						7.415				
2.0										
3.0										
4.0										
5.0										
6.0										
7.0										
8.0										
9.0										

Figure 1.11 Tables of cubes

Note that the steps (i) to (iii) are often shortened saying “1.9 under 5”

Mostly the values obtained from the table of cubes are only a approximate values which of course serves almost for all practical purposes.

Group work 1.5

Find the cube of the number 7.89.

a) use rough calculation method.

b) use the numerical table.

c) by calculating the exact value of the number.

d) compare your answer from “a” to “c”.

e) write your generalization.

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Example 25: Find the cube of the number 6.95.

Solution:

Do rough calculation and compare your answer with the value obtained from the table.

i. Rough Calculation

$6.95 \approx 7$ and $7^3 = 343$

$(6.95)^3 \approx 343$

ii. Value Obtained from the Table

Step i. Find the row which starts with 6.9

Step ii. Find the column head by 5

Step iii. Read the number, that is the intersection of the row in (i) and the column (ii), therefore $(6.95)^3 = 335.75$

iii. Exact Value

Multiply $6.95 \times 6.95 \times 6.95 = 335.702375$

so $(6.95)^3 = 335.702375$

This examples shows that the result obtained from the numerical tables is an approximation and more closer to the exact value.

Exercise 1I

1. Use the table of cubes to find the cube of each of the following.

a) 3.55 c) 6.58 e) 7.02 g) 9.86 i) 9.90 k) 9.97

b) 4.86 d) 6.95 f) 8.86 h) 9.88 j) 9.94 l) 9.99

1.3.2 Cube Root of a Number

Group work 1.6

Discuss with your group.

1. **In Figure 1.12 to the right, the volume of a cube is 64 m^3. What is the length of each edge?**
2. **Can you define a "cube root" of a number precisely by your own word?**
3. **Find the cube root of 12,167 × 42,875.**

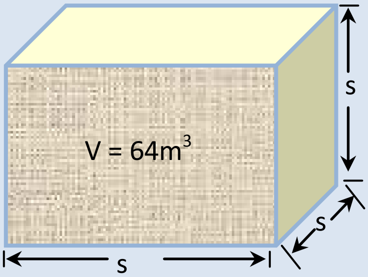

Figure 1.12 Cube

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Definition 1.6: **The cube root of a given number is one of the three identical factors whose product is the given number.**

Example 26:

a) $0\times 0\times 0 = 0$, so 0 is the cube root of 0.

b) $5\times5\times5 = 125$, so 5 is the cube root of 125.

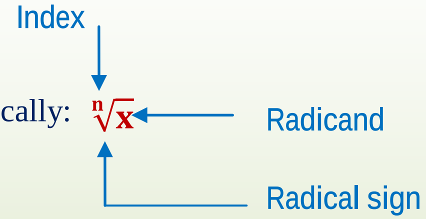
c) $\frac{1}{5}\times\frac{1}{5}\times\frac{1}{5}=\frac{1}{125}$, so $\frac{1}{5}$ is the cube root of $\frac{1}{125}$.

Note: i) $4^3 = 64$, 64 is the cube of 4 and 4 is the cube root of 64

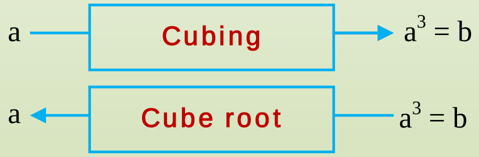
i.e $\sqrt[3]{64} = 4$.

ii) $\sqrt{\ }$ is a radical sign.

Or symbolically: $\sqrt[n]{x}$



iii) The relation of cubing and extracting cube root can be expressed as follows:



iv) a is the cube root of b and written as $a = \sqrt[3]{b}$.

When no index is written, the radical sign indicates a square root.

For example $\sqrt[3]{512}$ is read as "the cube root of 512".

The number 3 is called **the index** and 512 is called **the radicand**.

Cube Roots of Perfect Cubes

Group work 1.7

Discuss with your group

1. Find the cube root of the perfect cubes.

a) $\sqrt[3]{27}$ b) $\sqrt[3]{\frac{1}{27}}$ c) $\sqrt[3]{125}$ d) $\sqrt[3]{-64}$

2. Which of the following are perfect cubes?

{42,60,64,90,111,125,133,150,216}

3. Which of the following are perfect cubes?

$\{3,6,8,9,12,27,y^3, y^8, y^9, y^{12}, y^{27}\}$

Note: The following properties of cubes are important: $(ab)^3 = a^3 \times b^3$ and $\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$ (where $b \neq 0$).

Thus $(2 \times 2)^3 = 2^3 \times 2^3 = 8 \times 8 = 64$ and $\left(\frac{3}{4}\right)^3 = \frac{3^3}{4^3} = \frac{27}{64}$.

A number is called a perfect cube, if it is the cube of a rational number.

Definition 1.7: A rational number x is called a perfect cube if and only if $x = n^3$ for some $n \in \mathbb{Q}$.

Example 27:

$1 = 1^3$, $8 = 2^3$, $27 = 3^3$, $64 = 4^3$ and $125 = 5^3$.

Thus 1, 8, 27 , 64 and 125 are **perfect cubes.**

Note: A perfect cube is a number that is a product of three identical factors of a rational number and its cube root is also a rational number.

Example 28: Find the cube root of each of the following.

a) 216 b) $\frac{1}{8}$ c) -64 d) -27

Solution:

a) $\sqrt[3]{216} = \sqrt[3]{6\times6\times6} = 6$

c) $\sqrt[3]{-64} = \sqrt[3]{-4\times-4\times-4} = -4$

b) $\sqrt[3]{\frac{1}{8}} = \sqrt[3]{\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}} = \frac{1}{2}$

d) $\sqrt[3]{-27} = \sqrt[3]{-3\times-3\times-3} = -3$ -3

Exercise 1J

1. Determine whether each of the following statements is true or false.

a) $\sqrt[3]{17281} = 26$ b) $\sqrt[3]{\frac{1}{729}} = \frac{1}{90}$ c) $\sqrt[3]{-64} = \pm4$ d) $\sqrt[3]{\frac{-1}{625}} = \frac{1}{20}$

2. Find the cube root of each of the following.

a) 0 c) 1000 e) $\frac{1}{729}$ g) $\sqrt[3]{x^3}$ i) $\frac{-1}{1331}$

b) 343 d) 0. 001 f) $\frac{-1}{9261}$ h) $\frac{0}{27}$

3. Evaluate each of the following.

a) $\sqrt[3]{-27}$ d) $-\sqrt[3]{\frac{1}{64}}$ f) $\sqrt[3]{-1000}$ h) $-\sqrt[3]{-64}$

b) $\sqrt[3]{\frac{1}{8}}$ e) $\sqrt[3]{\frac{-8}{27}}$ g) $\sqrt[3]{\frac{-27}{64}}$ i) $\sqrt[3]{\frac{-1}{125}}$

c) $\sqrt[3]{27}$

Challenge Problem

4. Simplify: a) $5\sqrt{18} - 3\sqrt{72} + 4\sqrt{50}$ b) $\frac{2\sqrt{5}\times7\sqrt{2}}{\sqrt{14}\times\sqrt{45}}$

5. Simplify the expressions. Assume all variables represent positive rational number.

a) $\sqrt[3]{\frac{y^5}{27y^3}}$ c) $\sqrt[3]{16a^3}$ e) $\sqrt[3]{\frac{x^5}{x^2}}$ g) $\sqrt[3]{\frac{y^{11}}{y^2}}$

b) $\sqrt[3]{16z^3}$ d) $\sqrt[3]{\frac{b^4}{27b}}$ f) $\sqrt[3]{15m^4n^{22}}$ h) $\sqrt[3]{20s^{15}t^{11}}$

Table of Cube Roots

The same table which you can used to determine cubes of numbers can be used to find the approximate cube roots, of numbers.

Example 29: Find $\sqrt[3]{64.48}$ from the numerical table.

Solution: Find the value using rough calculations.

$$64.48 \approx 64; \sqrt[3]{64.48} \approx \sqrt[3]{64}$$

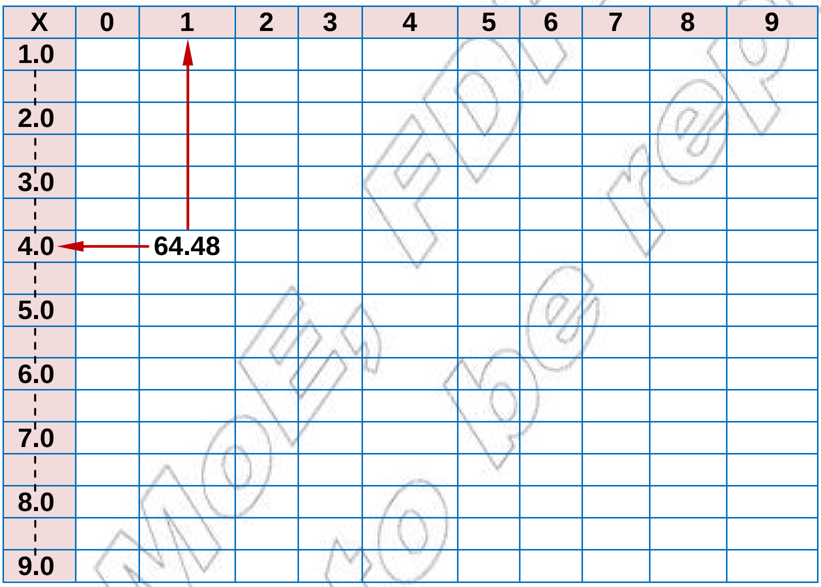
$$\approx \sqrt[3]{4 \times 4 \times 4} = 4$$

***Step i*:** Find the number 64. 48 in the body of the table for the relation $y = x^3$.

***Step ii*:** Move to the left on the row containing this number to get 4.0 under x. These are the first two digits of the required cube root of 64. 48.

***Step iii*:** To get the third digits start from 64.48 and move vertically upward and read 1 at the top.

There fore $\sqrt[3]{64.48} \approx 4.01$.



X	0	1	2	3	4	5	6	7	8	9
1.0										
2.0										
3.0										
4.0		64.48								
5.0										
6.0										
7.0										
8.0										
9.0										

Figure 1.13 Tables of cube roots

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Example 30:

In Figure 1.14 below, find the exact volume of the boxes.

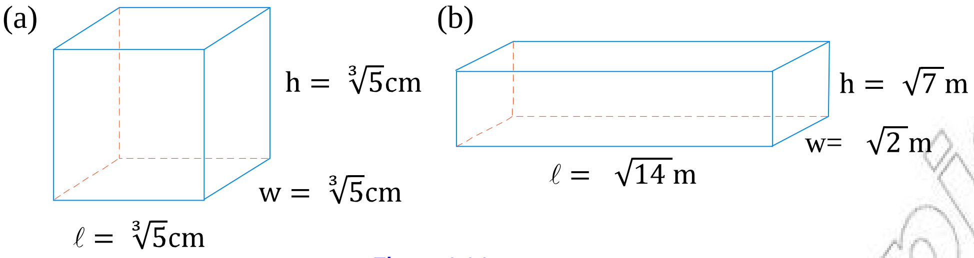

Figure 1.14

Solution

a) $V = \ell \times w \times h$

But the box is a cube, all the side of a cube are equal.

i.e $\ell = w = h = s$

$$V = s \times s \times s = s^3$$

$$V = \sqrt[3]{5}\text{cm} \times \sqrt[3]{5}\text{cm} \times \sqrt[3]{5}\text{cm}$$

$$V = \left(\sqrt[3]{5\text{cm}}\right)^3$$

$$V = \left(5^{\frac{1}{3}}\right)^3$$

$$V = 5\ \text{cm}^3$$

Therefore, the volume of the box is 5cm^3.

b) $V = \ell \times w \times h$

$$V = \sqrt{14}\text{m} \times \sqrt{2}\text{m} \times \sqrt{7}\text{m}$$

$$V = \sqrt{14}\text{m} \times \sqrt{14}\text{m}^2$$

$$V = \left(\sqrt{14 \times 14}\right)\text{m}^3$$

$$V = 14\text{m}^3$$

Therefore, the volume of the box is 14m^3.

Exercise 1k

1. Use the table of cube to find the cube root of each of the following.

a) 32.77 c) 302.5 e) 3114

b) 42.6 d) 329.5 f) 3238

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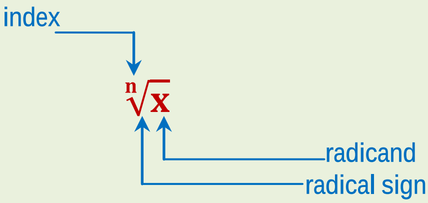
Summary for unit 1

1. *The process of multiplying a number by itself is called* **squaring** *the number.*
2. *For each rational number x there is a rational number y* $(y \geq 0)$ *such that* $x^2 = y$.
3. *A square root of a number is one of its two equal factors.*
4. *A rational number x is called a* **perfect square,** *if and only if* $x = n^2$ *for some* $n \in \mathbb{Q}$.
5. *The process of multiplying a number by itself three times is called* **cubing** *the number.*
6. *The cube root of a given number is one of the three identical factors whose product is the given number.*
7. *A rational number x is called a* **perfect cube,** *if and only if* $x = n^3$ *for some* $n \in \mathbb{Q}$.
8. 

9. *The relationship of squaring and square root can be expressed as follows:*

$$a \xrightarrow{\text{Squaring}} a^2 = b \text{ where } b \geq 0$$

$$a \xleftarrow{\text{Square root}} a^2$$

- *a is the square root of b and written as* $a = \sqrt{b}$

10. *The relationship of cubing and cube root can be expressed as follows:*

$$a \xrightarrow{\textbf{Cubing}} a^3 = b$$

$$a \xleftarrow{\textbf{Cube root}} a^3 = b$$

- *a is the cube root of b and written as* $a = \sqrt[3]{b}$

Miscellaneous Exercise 1

1. Determine whether each of the following statements is true or false.

a) $\frac{3\sqrt{8}}{2\sqrt{32}} = \frac{-3}{4}$

b) $\sqrt{7\frac{1}{9}} = \sqrt{\frac{64}{9}}$

c) $\sqrt{\frac{2}{5}}\sqrt{\frac{125}{8}} = 2.5$

d) $\sqrt{\frac{1}{7}} \times \sqrt{63} = \pm 3$

e) $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

f) $\sqrt{0.25} = \frac{-1}{2}$

g) $\sqrt{0.0036} = 0.06$

2. Simplify each expression.

a) $\sqrt{\frac{36}{324}}$

b) $\frac{\sqrt{50}}{\sqrt{2}}$

c) $8\sqrt{\frac{25}{4}}$

d) $\sqrt{\frac{16}{4}}$

e) $2\sqrt{2}\left[\frac{3}{\sqrt{2}} + \sqrt{2}\right]$

3. Simplify each expression.

a) $\sqrt{600}$

b) $\sqrt{50} + \sqrt{18}$

c) $\left(5\sqrt{6}\right)^2$

d) $\sqrt{3}\left(\sqrt{3} + \sqrt{6}\right)$

e) $\sqrt{19^2}$

f) $\sqrt{64 + 36}$

g) $\sqrt{2}\left(\sqrt{2} + \sqrt{6}\right)$

h) $\sqrt{2}\left(\sqrt{3} + \sqrt{8}\right)$

4. Simplifying radical expressions (where x ≠ 0).

a) $\frac{\sqrt[3]{32}}{\sqrt[3]{-4}}$

b) $\frac{\sqrt[3]{162x^5}}{\sqrt[3]{3x^2}}$

c) $\frac{\sqrt{12x^4}}{\sqrt{3x}}$

d) $\sqrt[3]{80n^5}$

e) $\sqrt[3]{p^{17}q^{18}}$

5. Study the pattern and find a and b

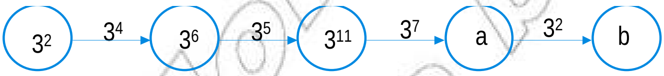


Figure 1.15

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6. Study the pattern and find a, b, c and d.

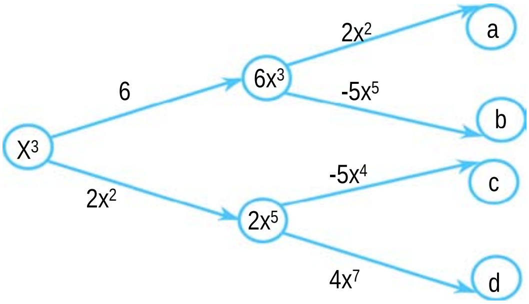

Figure 1.16

7. An amoeba is a single cell animal. When the cell splits by a process called “fission” there are then two animals. In a few hours a single amoeba can become a large colony of amoebas as shown to the right.

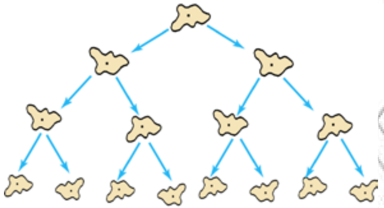

Figure 1.17

Number of splits	Number of amoeba cells
0	1
1	2
2	$4 = 2 \times 2 = 2^2$
3	$8 = 2 \times 2 \times 2 = 2^3$

How many amoebas would there be

a) After 4 splits?
c) after 6 splits?

b) After 5 splits?
d) after 10 splits?

8. Using only the numbers in the circular table, write down, all that are:

a) square numbers

b) cube numbers

Figure 1.18

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9. Find the exact perimeter of a square whose side length is $5\sqrt[3]{16}$ cm.

10. The length of the sides of a cubes is related to the volume of the cube according to the formula: $x = \sqrt[3]{V}$.
   a) What is the volume of the cube if the side length is 25cm.
   b) What is the volume of the cube if the side length is 40 cm.

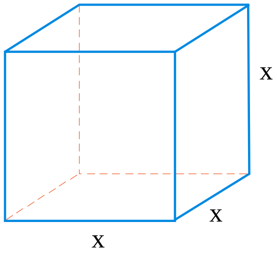

Figure 1.19 Cube

11. In Figure 1.20 to the right find:
   a) the surface area of a cube.
   b) the volume of a cube.
   c) compare the surface area and the volume of a given a cube

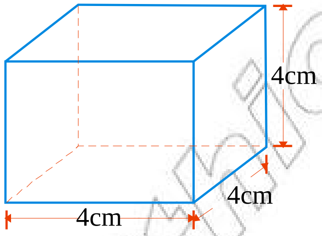

Figure 1.20 Cube

12. Prove that the difference of the square of an even number is multiple of 4.

13. Show that 64 can be written as either 2^6 or 4^3.

14. Look at this number pattern.

$7^2 = 49$
$67^2 = 4489$
$667^2 = 444889$
$6667^2 = 44448889$

This pattern continues.
   a) Write down the next line of the pattern.
   b) Use the pattern to work out 6666667^2.

15. Find three consecutive square numbers whose sum is 149.

16. Find the square root of $25x^2 - 40xy + 16y^2$.

17. Find the square root of $\frac{64a^2}{9b^2} + 4 + \frac{32a}{3b}$.

18. Find the cube root of $27a^3 + 54a^2b + 36ab^2 + 8b^3$.

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